Analysis of the BESSY II Results:
Bunch Distortion Effects

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The 'Famous' CSR Equation

$$P = p\left[N + N(N-1)g(\omega)\right]$$

$$g(\omega) = \left| \int_{-\infty}^{\infty} dz\, S(z) e^{i\omega\cos(\theta)z/c} \right|^2$$

Normalized Bunch Longitudinal Distribution

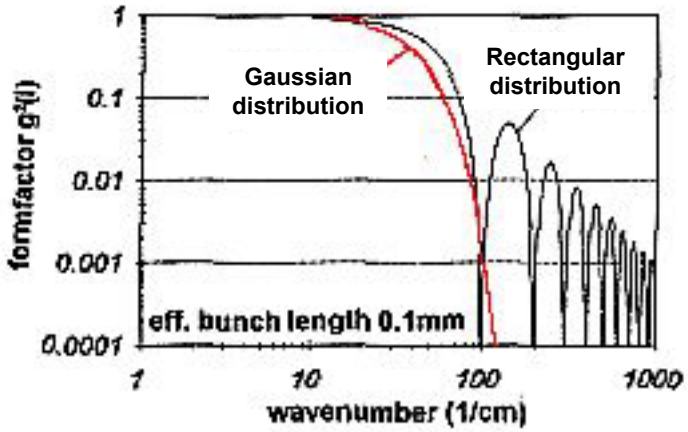


FIGURE 3. Form factor for a Gaussian and rectangular particle distribut

Two possible knobs for CSR production:

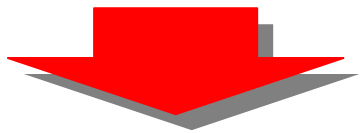
- **Shortening the Bunches**
- **'Distorting' the Bunches**

Steady CSR Emission at BESSY II

Low momentum compaction: $\alpha_C \sim 10^{-6} \div 10^{-5}$

Measured bunch length: $\sigma_z \sim 1 \div 1.5$ mm

CSR observed up to wavenumbers of ~ 50 cm^{-1} (λ = 200 μm)



CSR seems to be generated by non-gaussian bunches

M. Abo-Bakr et al., *Steady-State Far-Infrared Coherent Synchrotron Radiation detected at BESSY II,* Phys. Rev. Lett. 88, 254801 (2002)

BESSY II Streak Camera Measurements

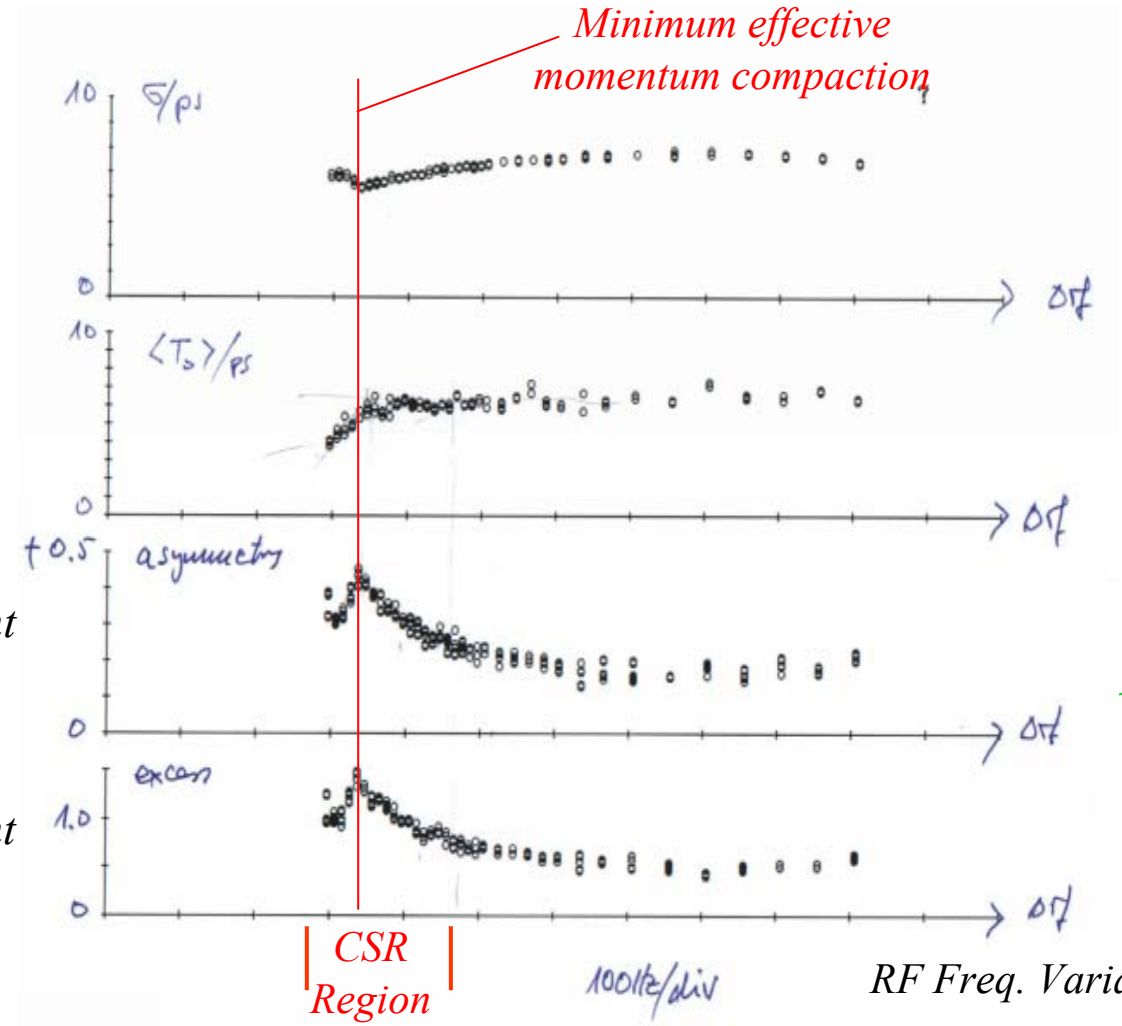
~ 2 μA/bunch

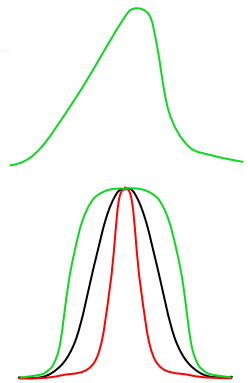
Rms length

Synchronous Phase

3^{rd} *Order Distr. Moment*

4^{th} *Order Distr. Moment*





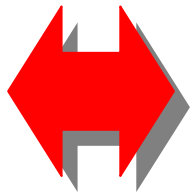
RF Freq. Variation

Possible Distorting Knobs

Non-linearities in the longitudinal dynamics:

~~**RF non-linearities**~~

Lattice non-linearities



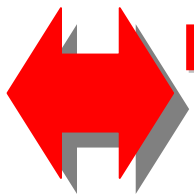
Requires only a finite momentum spread value

And/Or

Wakefields:

CSR Impedance

Vacuum Chamber Impedance (not included in this talk)



Requires some current to be effective

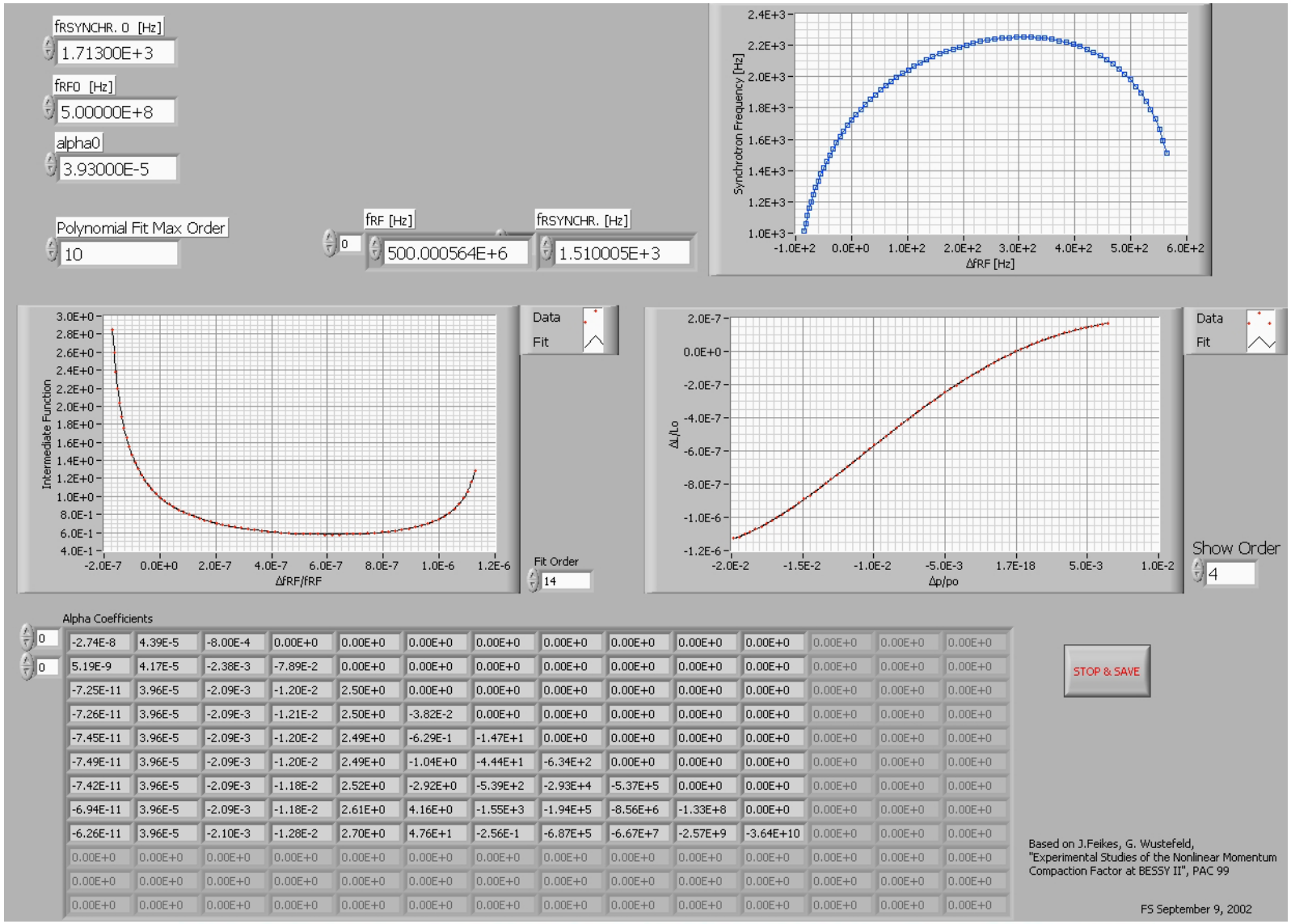
Lattice Non-Linearities

$$\frac{\Delta L}{L_0} = \alpha_0 (betatron motion) + \left(\alpha - \frac{1}{\gamma^2} \right) \frac{\Delta p}{p_0}$$

$$\alpha = \alpha_1 + \alpha_2 \frac{\Delta p}{p_0} + \alpha_3 \left(\frac{\Delta p}{p_0} \right)^2 + \ldots.$$

2-D simulations of the longitudinal dynamics

Measurement of the Momentum Compaction



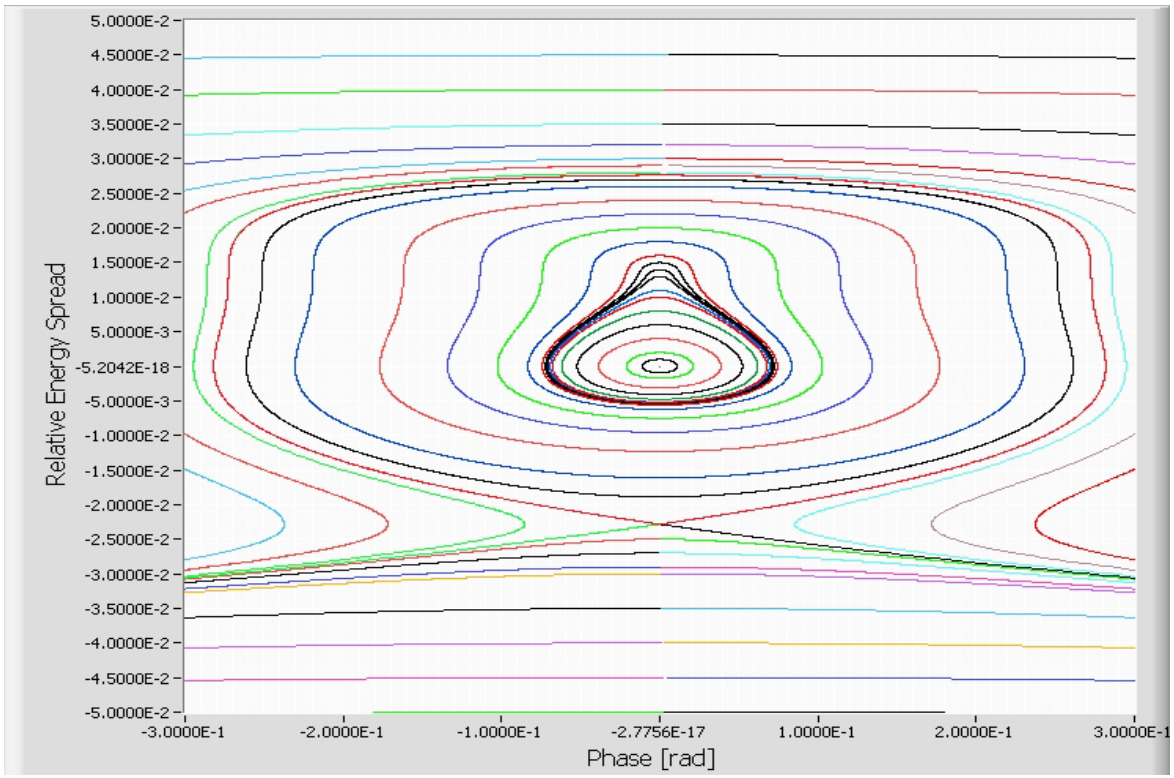
Momentum Compaction Terms Evaluated by Codes

Codes that allow using Differential Algebra (DA):

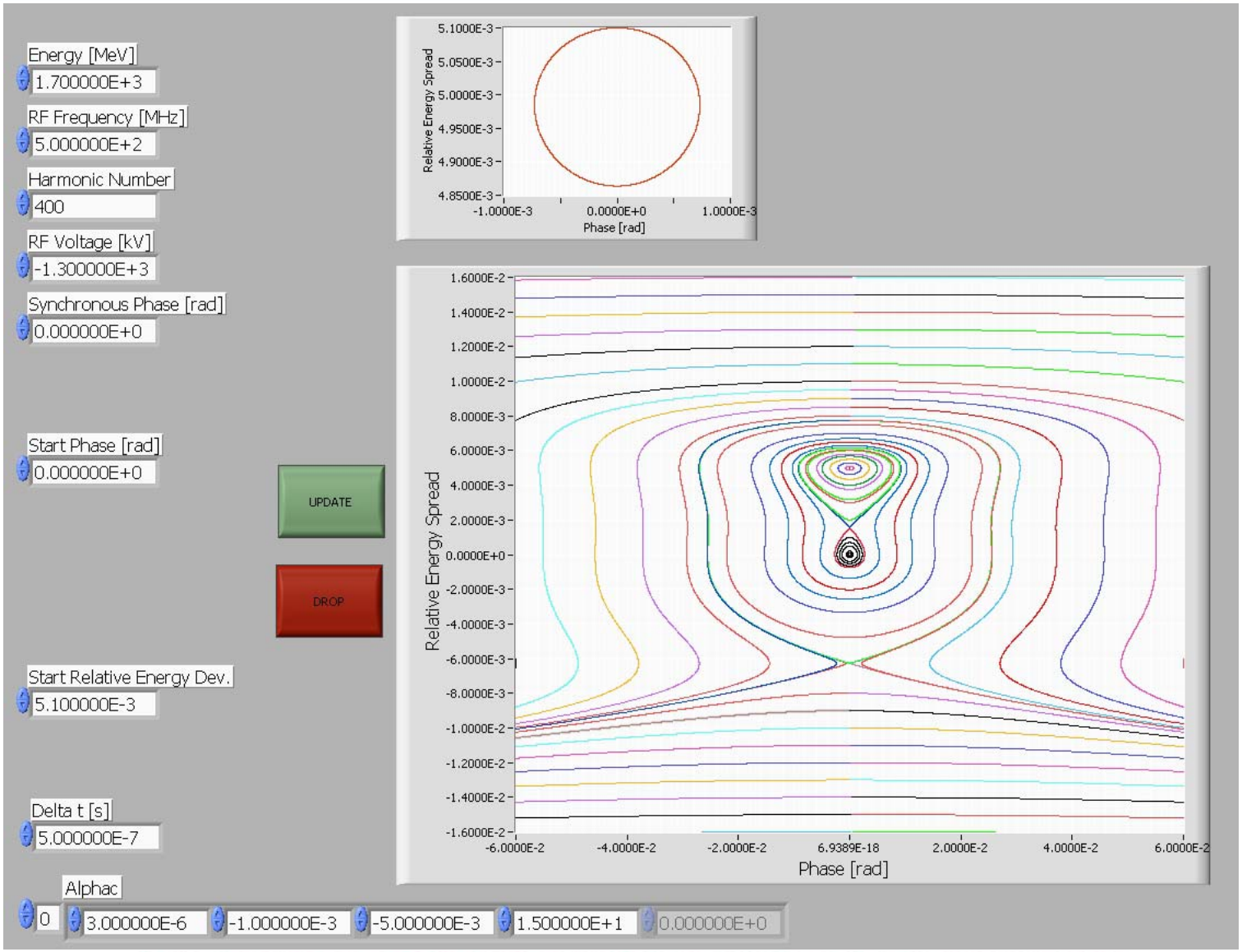
- Polymorphic Tracking Code (PTC)

- COSY-Infinity

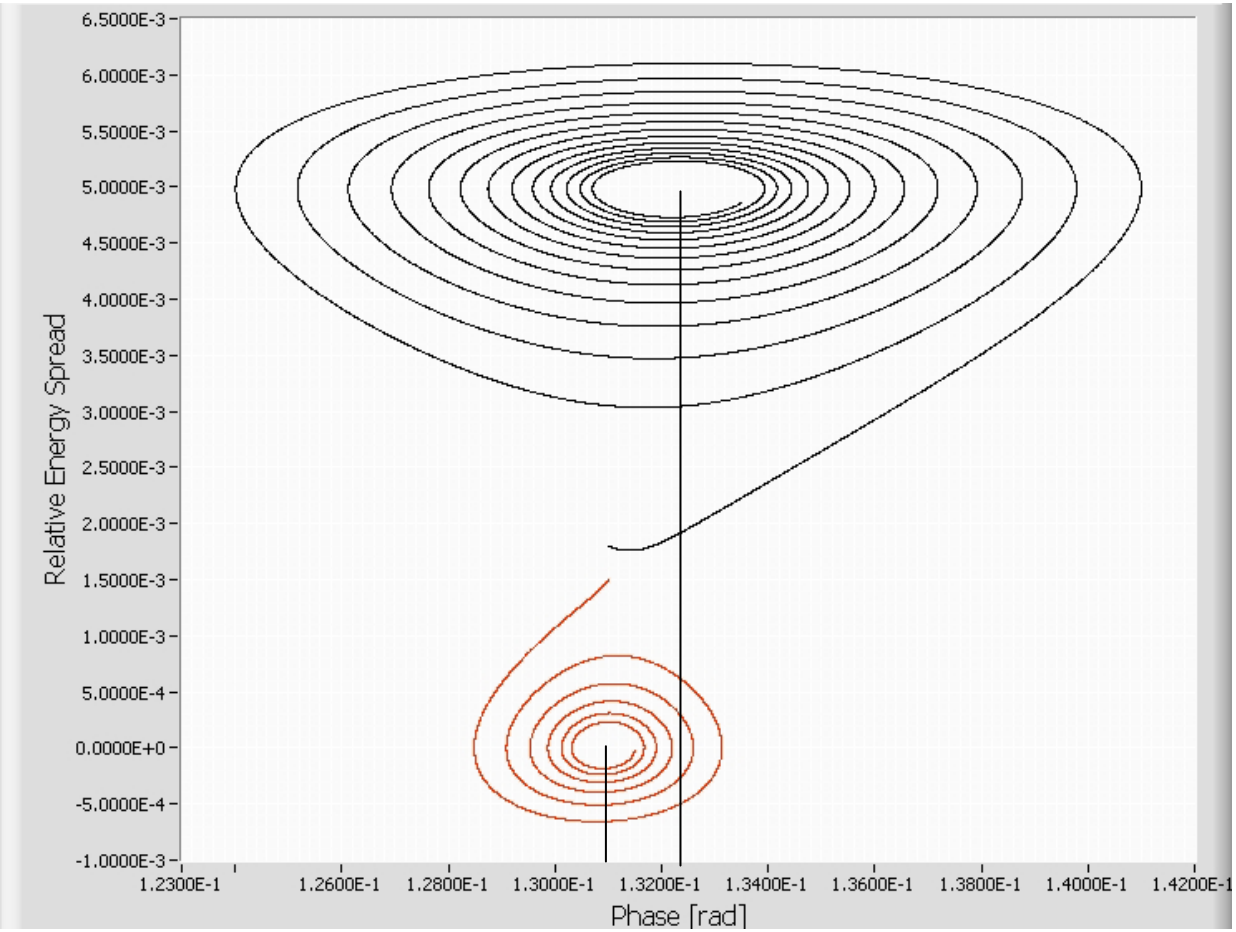
BESSY II Case Phase Space



A Particular Case

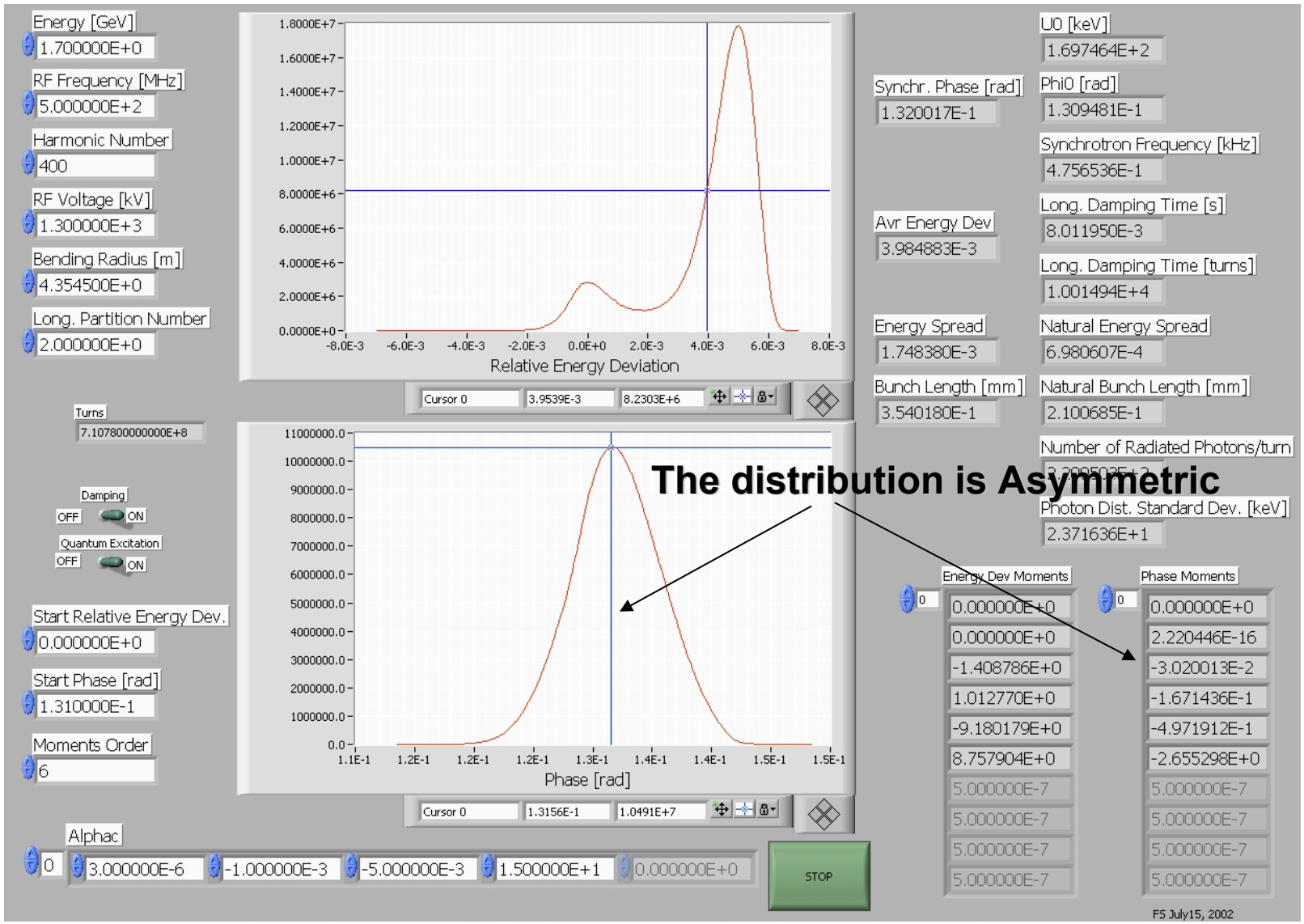


SR Energy Losses Effect



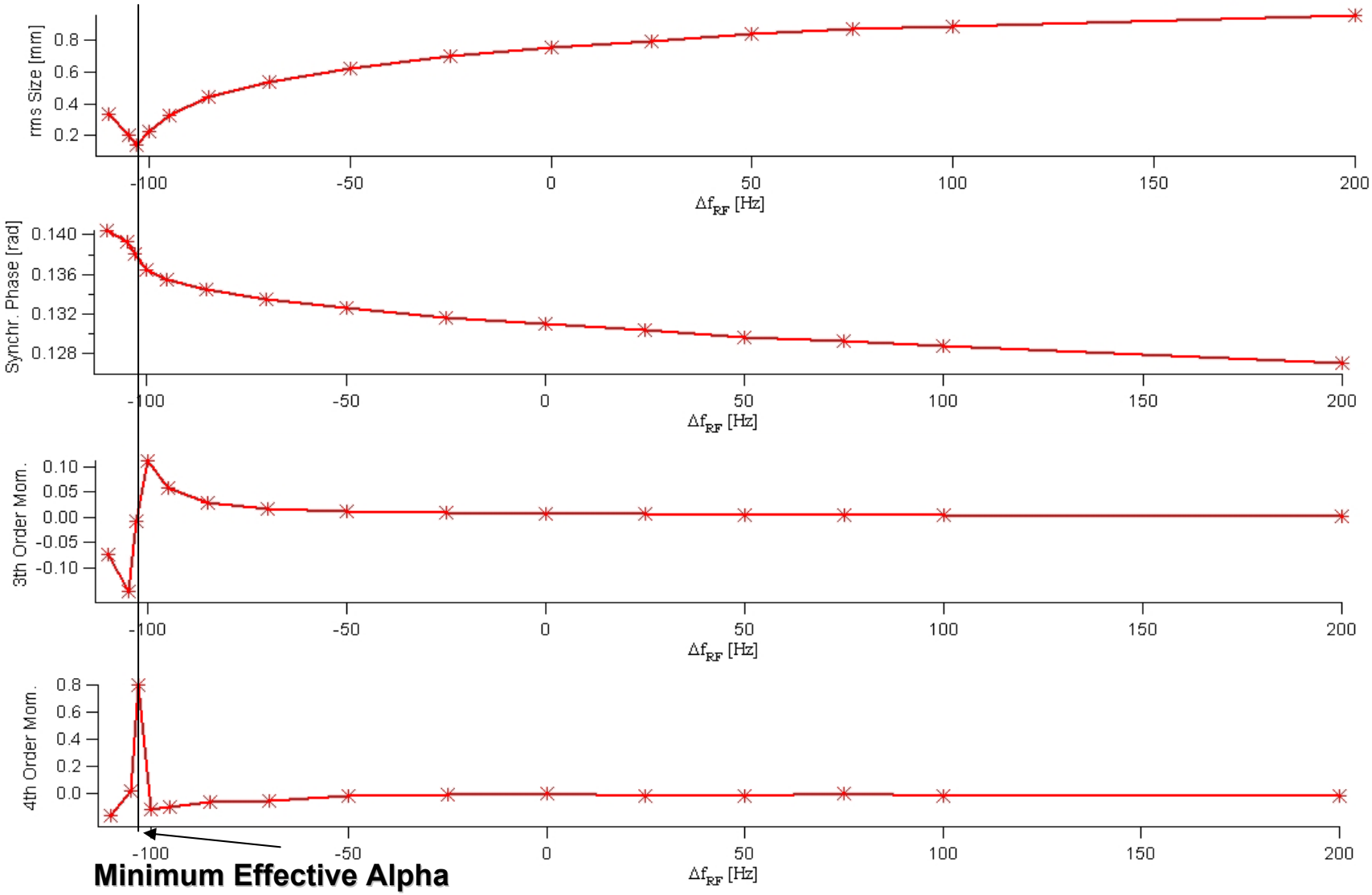
The SR energy losses break the symmetry of the phase plane.

Two Energy Buckets Simulation



The distribution is Asymmetric

Simulation of the BESSY II Results



Minimum Effective Alpha

BESSY II Streak Camera Measurements

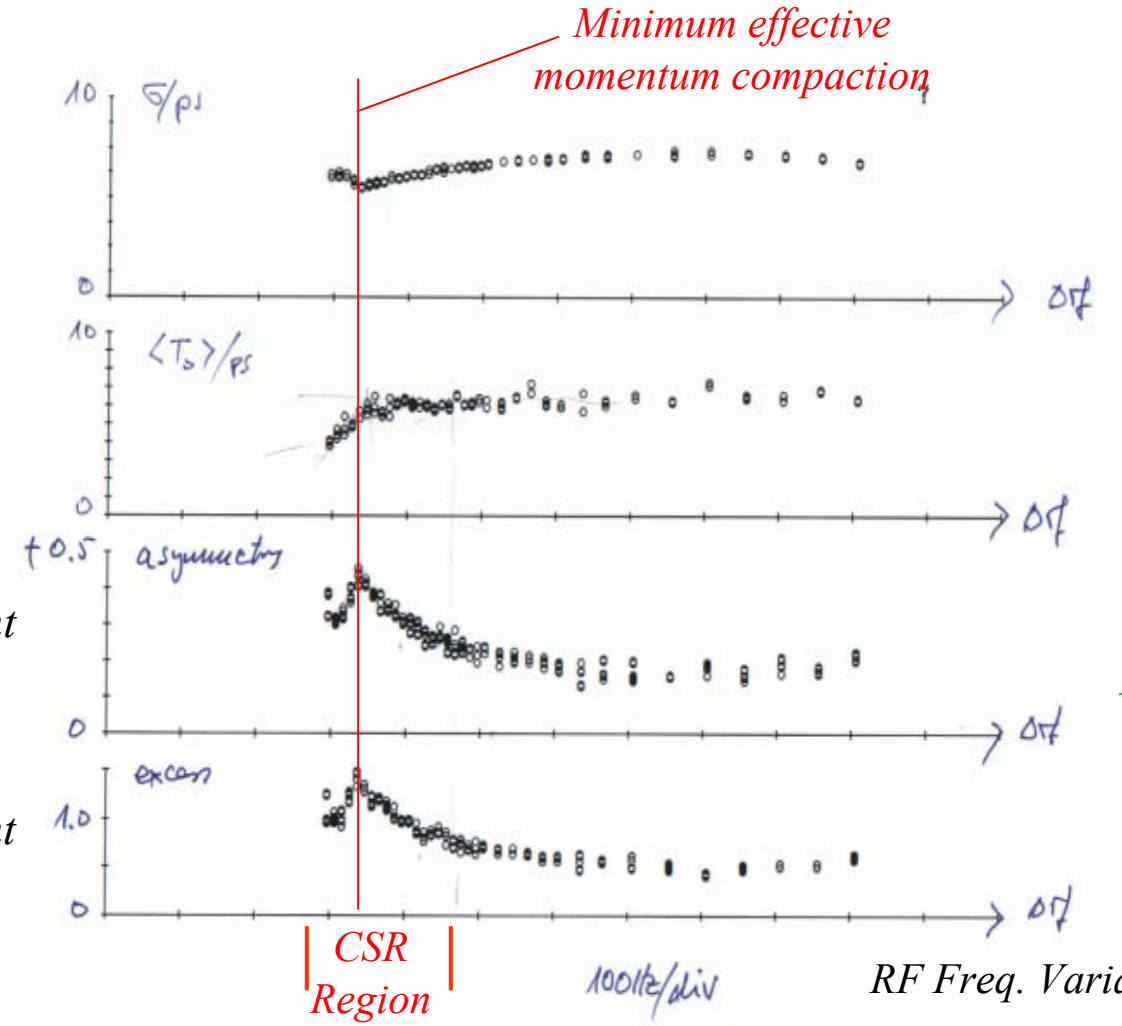
~ 2 μA/bunch

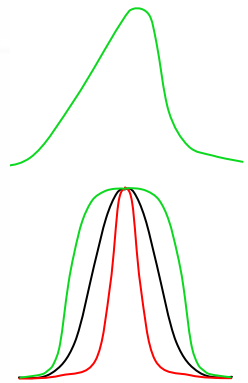
Rms length

Synchronous Phase

3^{rd} *Order Distr. Moment*

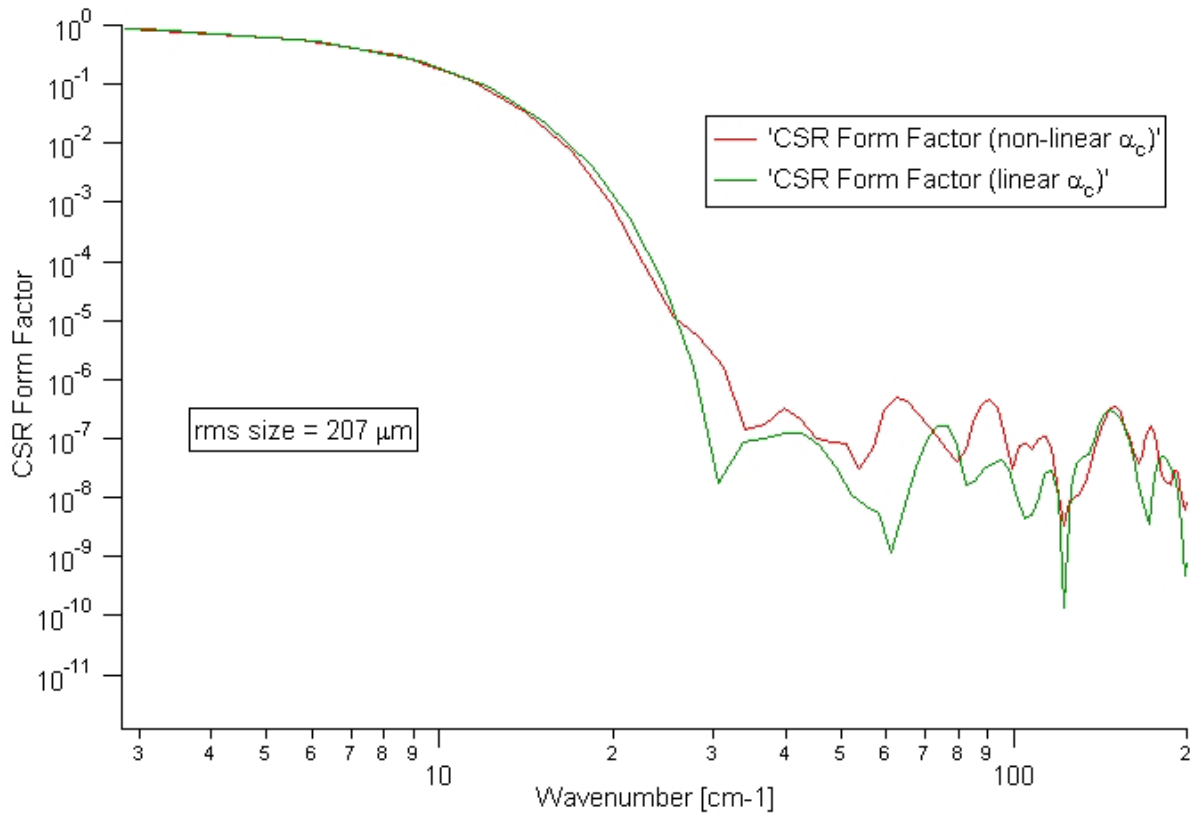
4^{th} *Order Distr. Moment*



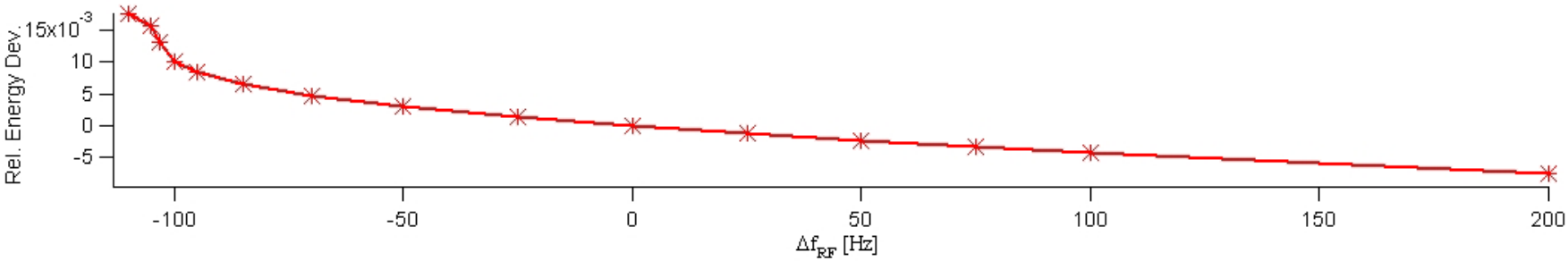


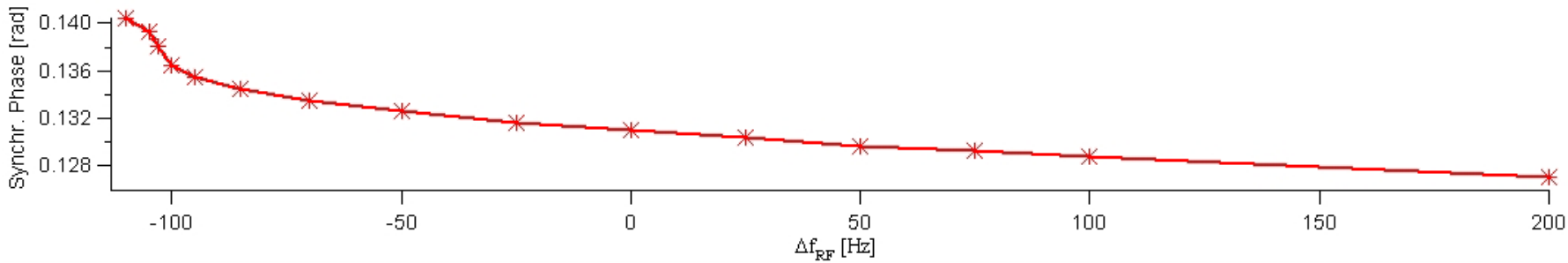
RF Freq. Variation

CSR Factor: Linear & Non-linear Cases



Relative Energy Deviation and Synchronous Phase





A Simple Model for the CSR Impedance

$$S(s) = -Z_0 \left(\frac{\rho}{3} \right)^{1/3} s^{-1/3}$$

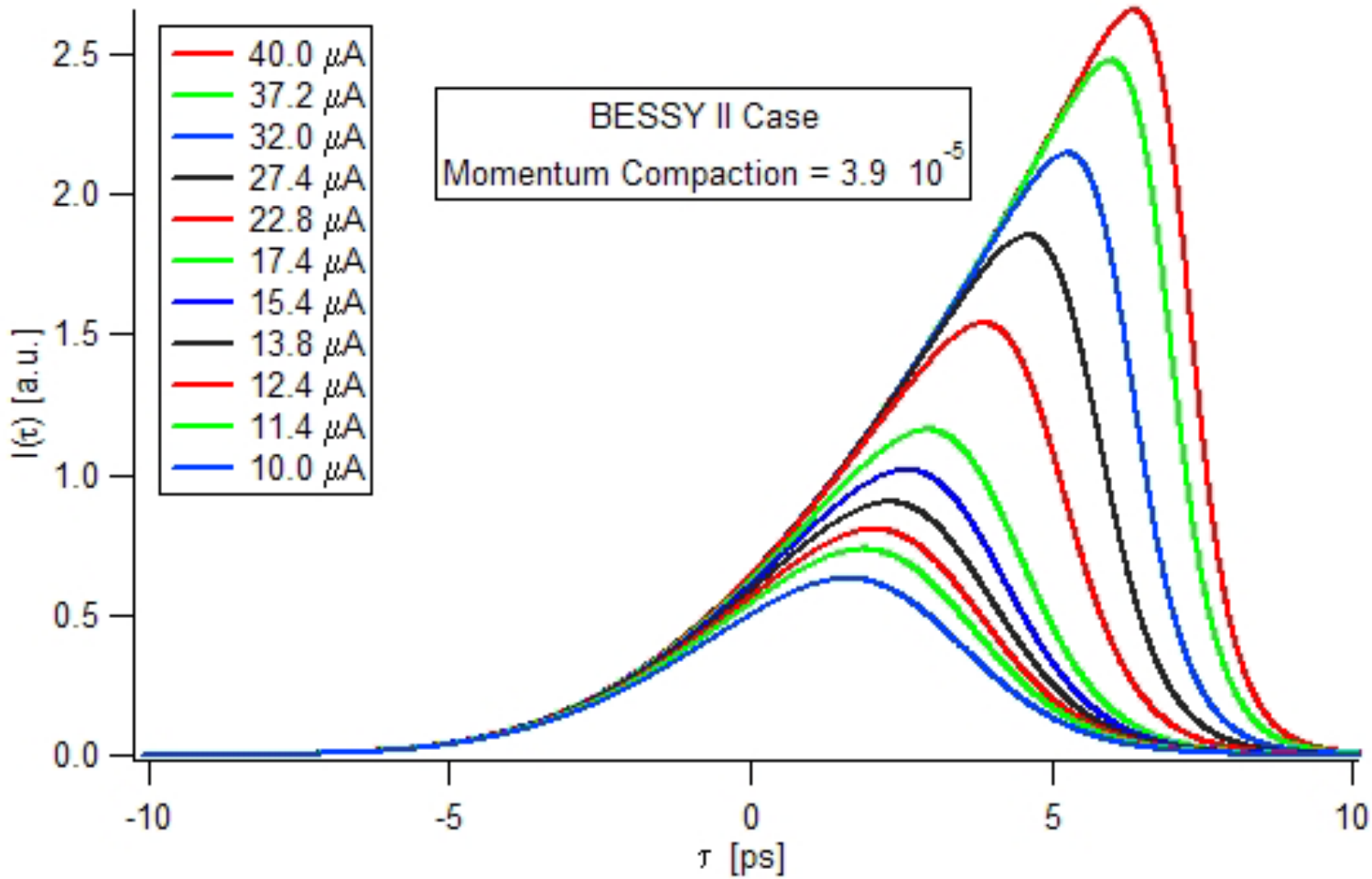
CSR Impedance for the Free Space

$$I(s) = \tilde{K}\, e^{-\frac{s^2}{2\sigma_0^2} - \frac{1}{\sigma_0^2 V'_{RF}} \int_0^\infty I(s-s')S(s')ds'}$$

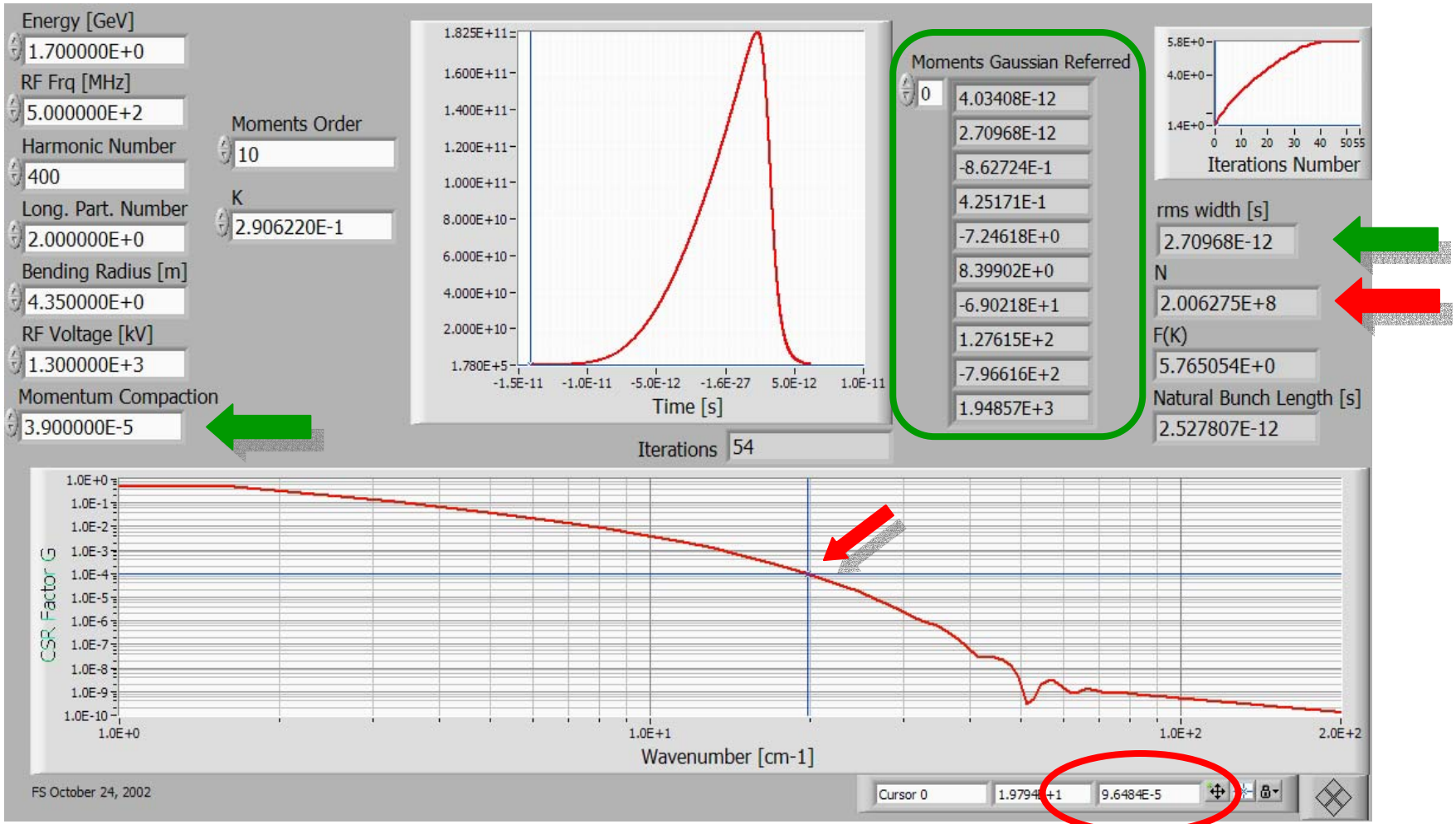
Haissinski Equation

K. Bane, S. Krinsky, J.B. Murphy, *Microbunches Workshop,* Upton NY 1995

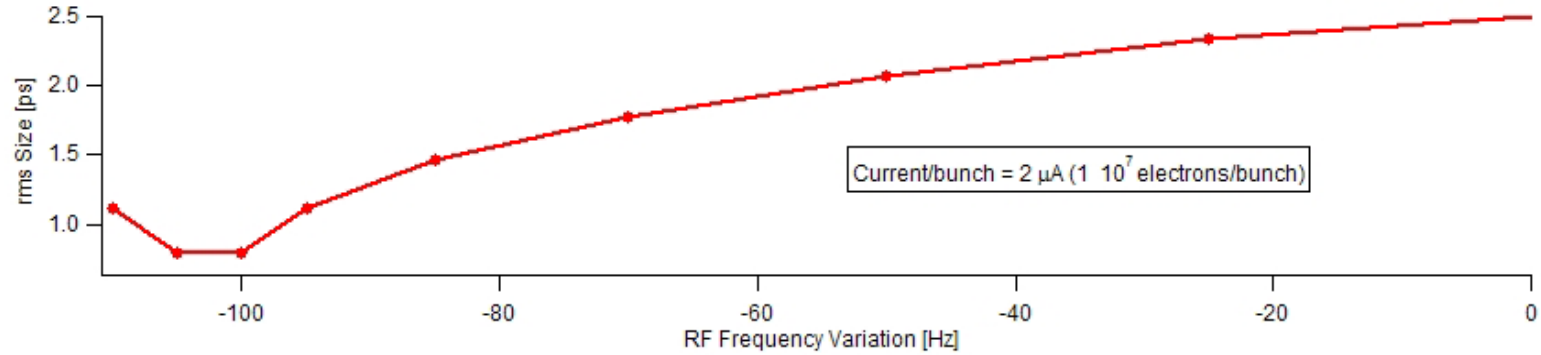
A Numerical Solution for The Equilibrium Distribution

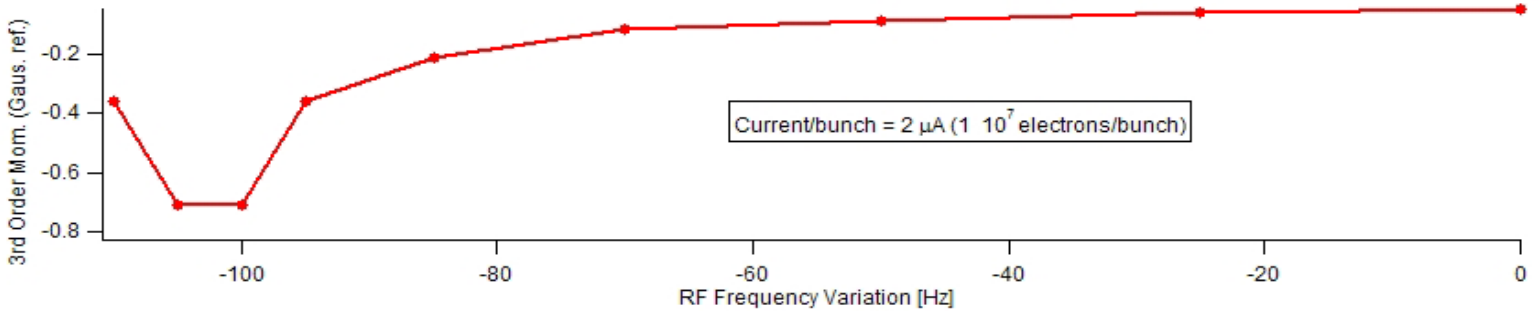


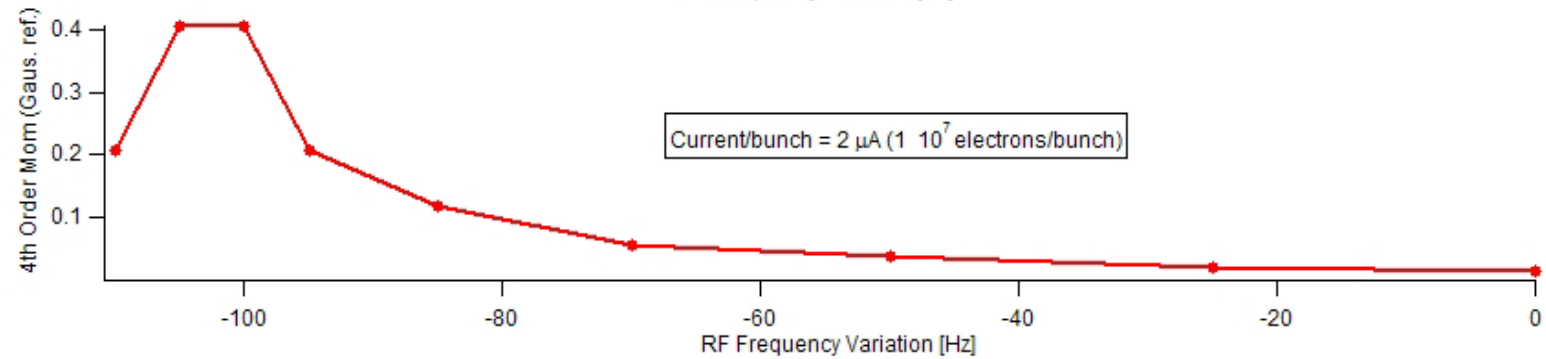
And From the Distribution...



Simulation of the BESSY II Results







BESSY II Streak Camera Measurements

~ 2 μA/bunch

Rms length

Synchronous Phase

3^{rd} *Order Distr. Moment*

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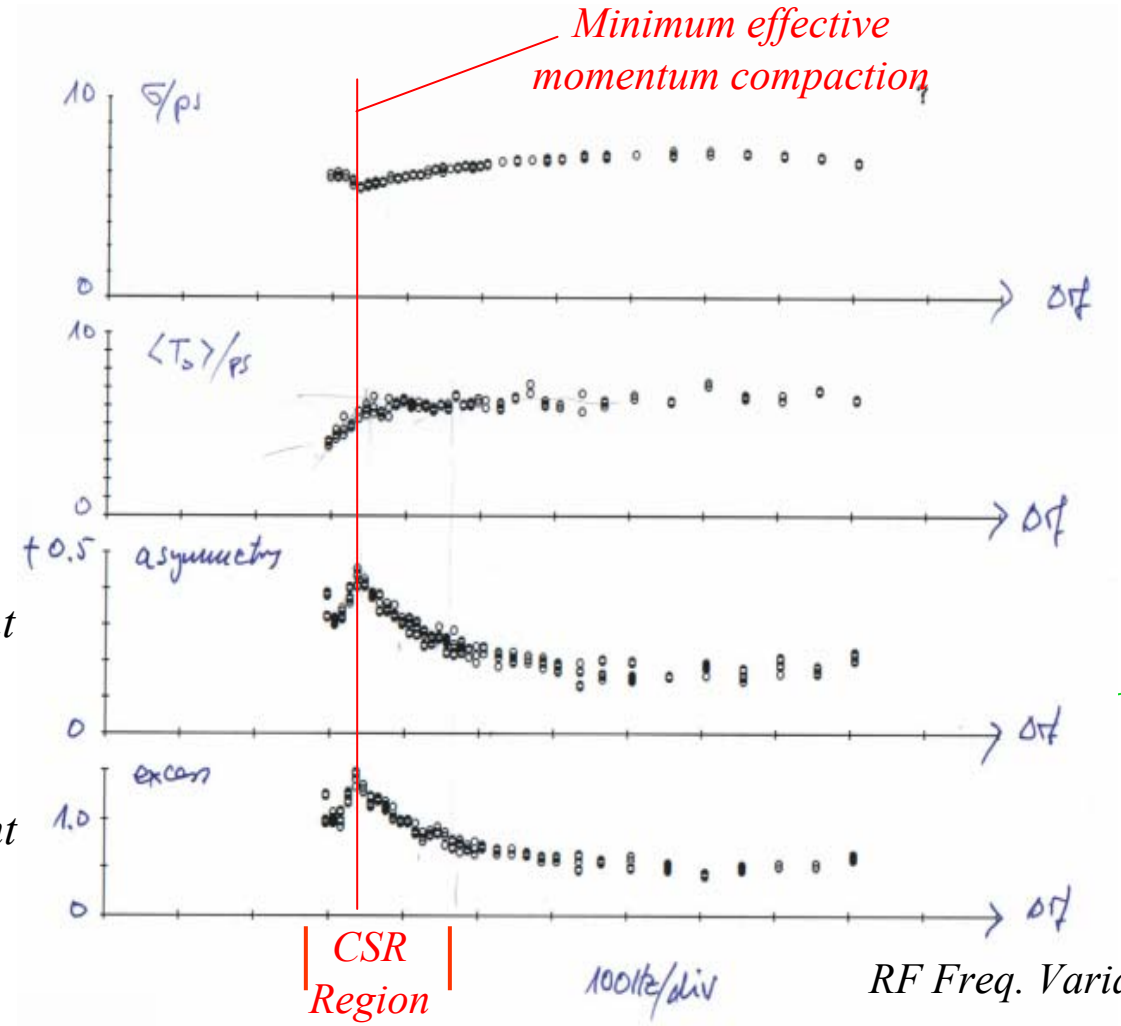
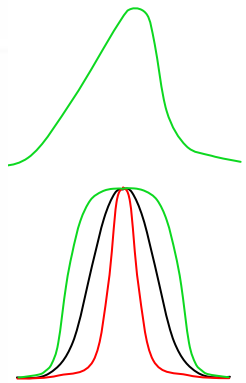
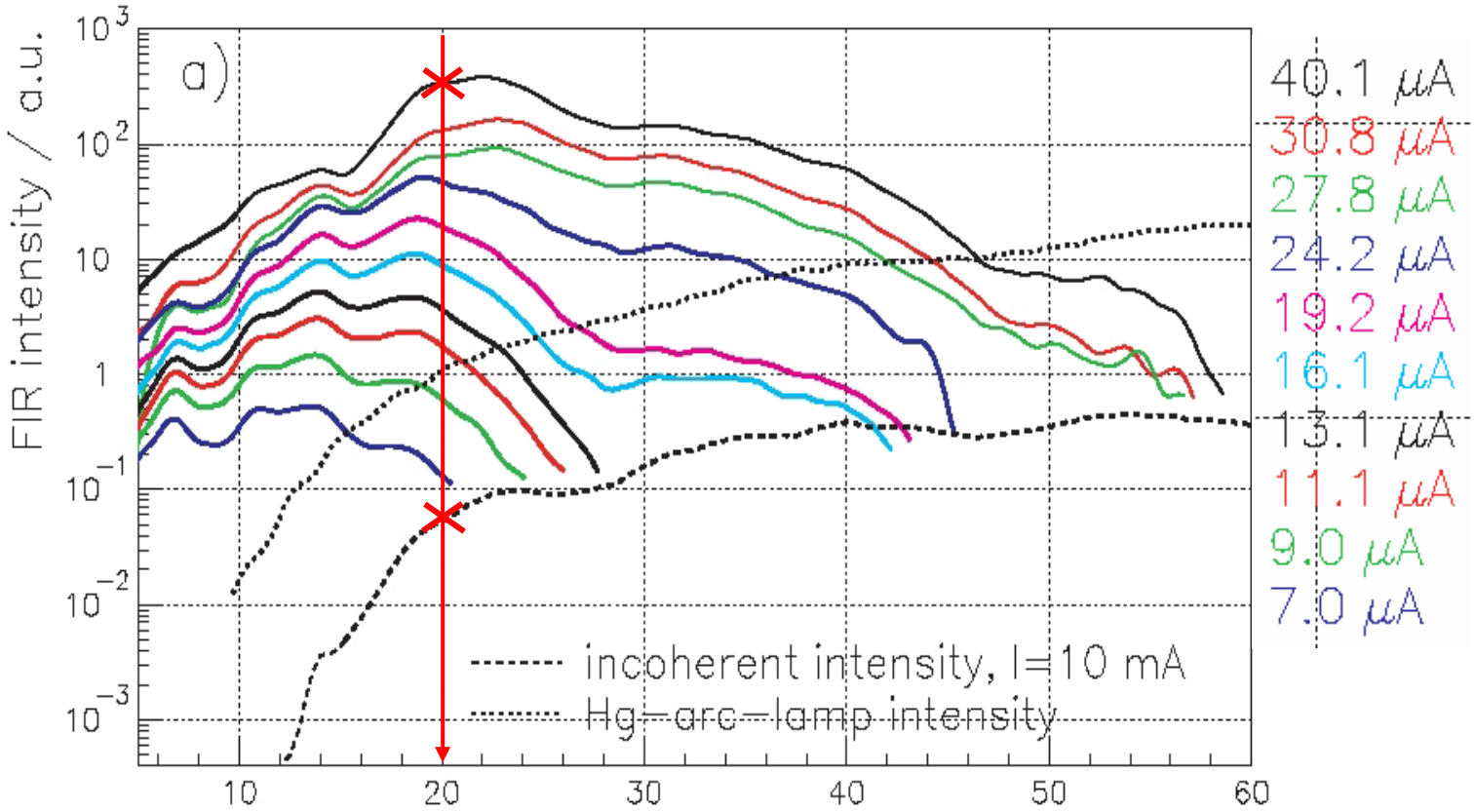




RF Freq. Variation

BESSY II CSR Spectra Measurement



@ 20 cm^{-1} and 40 μA

P_{CSR}/P_{SR} ~ 1.7 10^4

CSR Form Factor Value for BESSY II Case

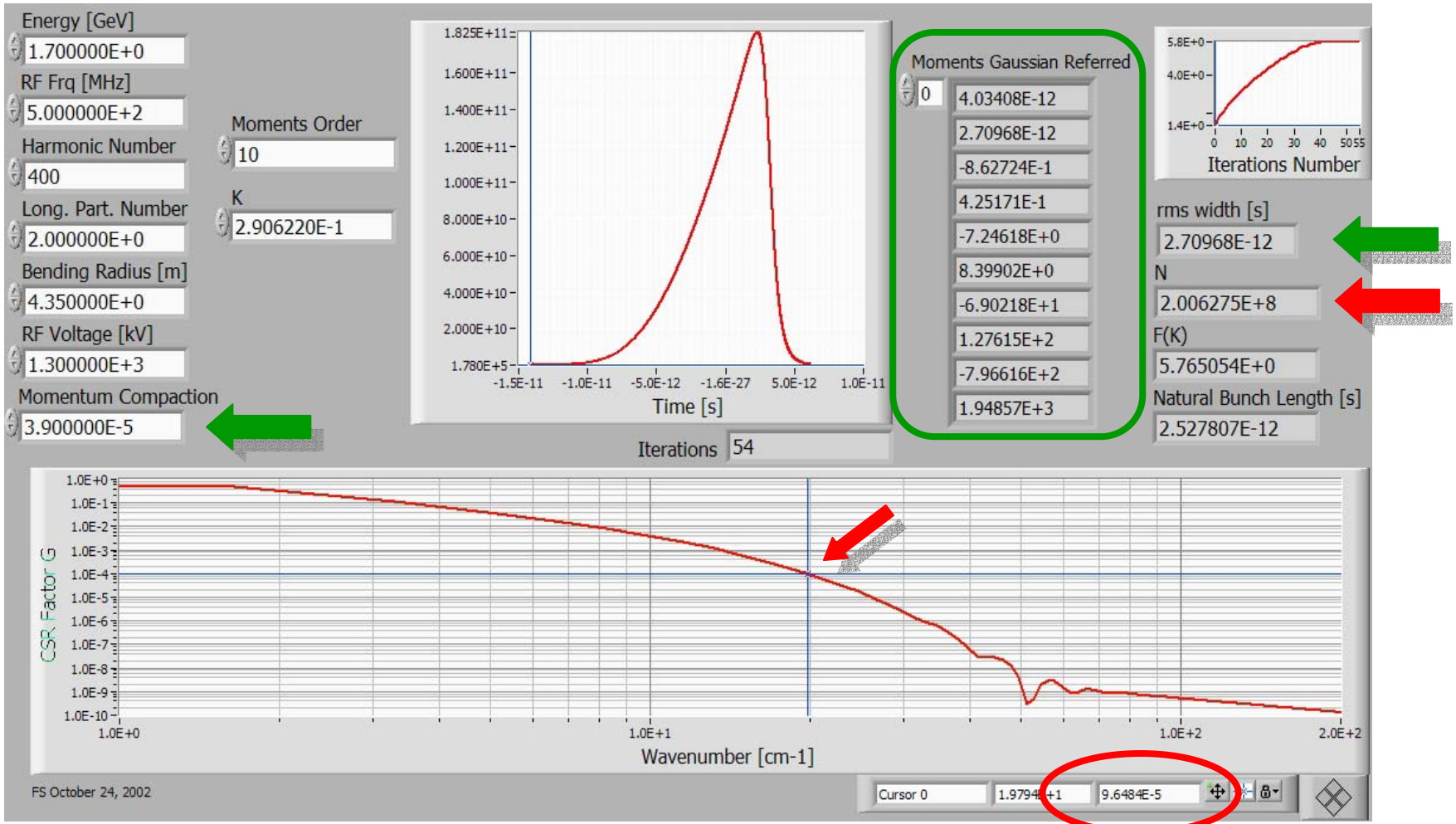
$$P = n_b p\left[N + N(N-1)g(\omega)\right]$$

$$\frac{P_{CSR}}{P_{SR}} \cong N g(\omega) \qquad if \quad P_{CSR} >> P_{SR}$$

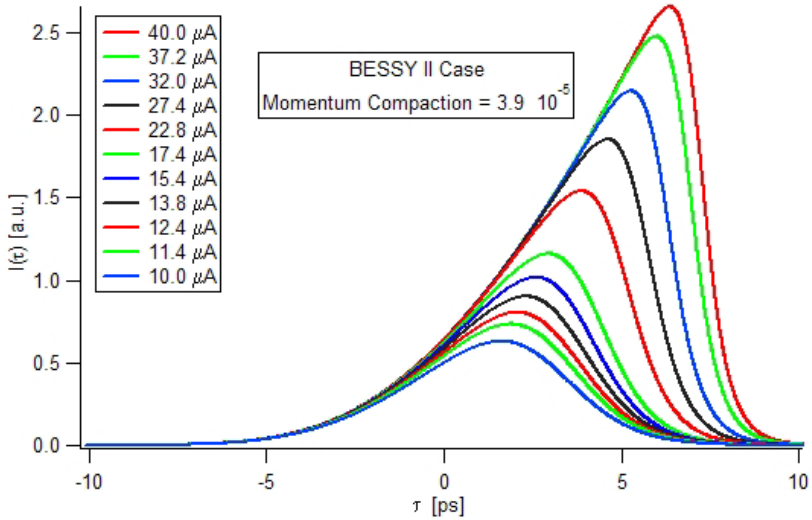
$$For \;\; I_b = 40 \;\; \mu A / bunch \qquad N = 2 \cdot 10^8 \qquad electrons / bunch$$

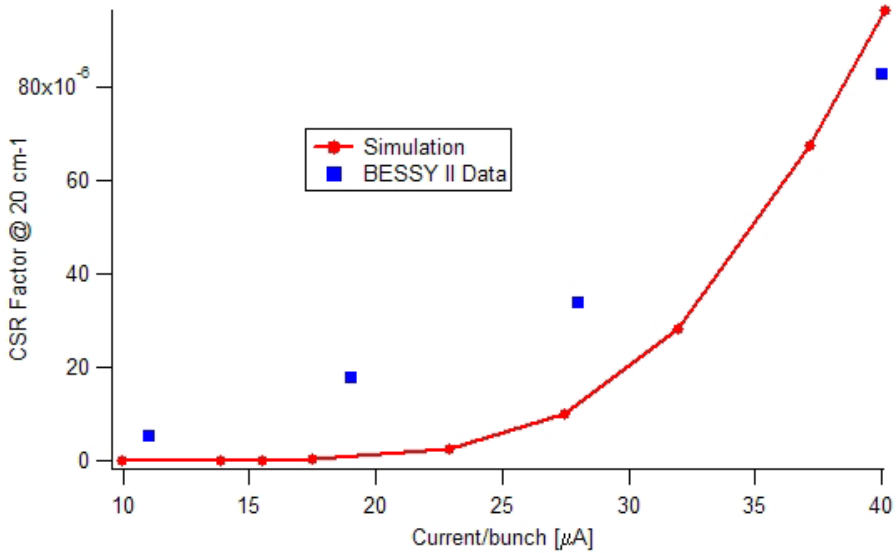
$$g(20\,cm^{-1}, 40\,\mu A) = \left.\frac{P_{CSR}}{P_{SR}}\right|_{20\,cm^{-1},\,40\,\mu A} \frac{1}{N_{40\,\mu A}} = 8.3 \cdot 10^{-5}$$

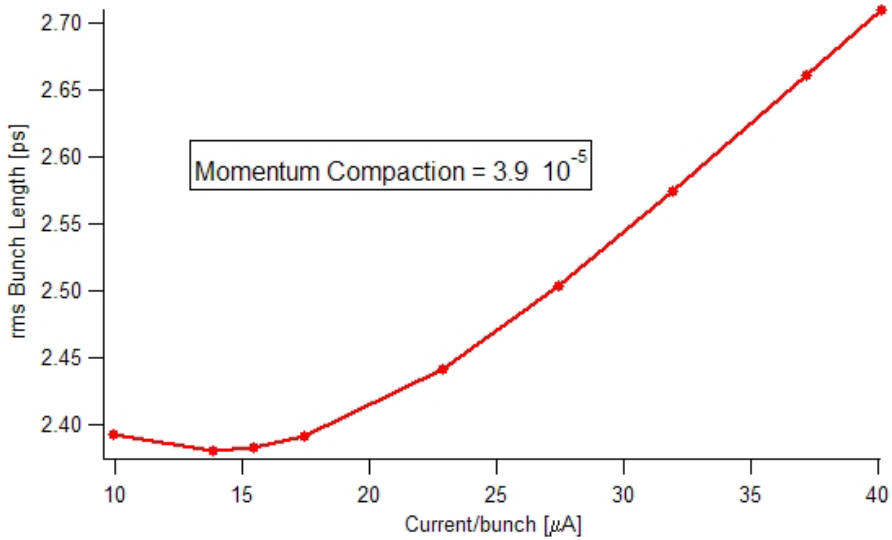
A Unique Solution



Simulation of the BESSY II Results





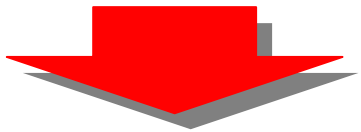


Final Considerations

The distortion due to lattice non-linearities is not sufficient to justify the CSR emission in the case of BESSY II.

A simple model for the CSR impedance allows to qualitatively explain most of the features of the BESSY II Results

The numerical results of the simulations are not so far from the measured values



The results indicate that the CSR wakefield plays a fundamental role

Still to be investigated:

CSR induced Instabilities

Effects of the energy dependent terms of the momentum compaction

Effects of the vacuum chamber: cut-off and impedance